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Certainty equivalent with normal truncated lotteries: an application for risk valuation of a health insurance policy in Mexico

Equivalente de certeza con loterías truncadas normales: una aplicación para la valuación de riesgo de una póliza de seguro de salud en México

Jorge O. Moreno*

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Abstract

This paper introduces a family of measures of welfare under uncertainty based on the certainty equivalent under the special case of normal truncated lotteries with constant risk aversion. It presents an application to the welfare gains from providing a public health insurance policy, and estimates the distribution of gains from reducing the risk in the households' net consumption. Using the Mexican National Household Income Expenditure Survey (ENIGH) for 2004, a calibration exercise is performed to test the model's implications. For a household with constant risk aversion/risk tolerance of 1 and facing a health expenditure shock with a mean-variance of $8-24 per quarter, the insurance policy would imply a consumption gain of 1.8 percent.

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Resumen

Este trabajo presenta una familia de medidas de bienestar bajo incertidumbre basadas en el equivalente de certeza en el caso especial de loterías truncadas normales con aversión al riesgo constante. Presenta una aplicación a las ganancias de bienestar derivadas de la provisión de una póliza pública de seguro de salud y estima la distribución de las ganancias derivadas de la reducción del riesgo en el consumo neto de los hogares. Utilizando la Encuesta Nacional de Ingresos y Gastos de los Hogares (ENIGH) de México de 2004, se realiza un ejercicio de calibración para evaluar las implicaciones del modelo. Para un hogar con una aversión al riesgo/tolerancia al riesgo constante de 1 y que enfrenta un shock de gasto en salud con una media-varianza de $8-24 por trimestre, la póliza de seguro implicaría una ganancia de consumo del 1.8%.

*Autor de correspondencia
Universidad Autónoma de Nuevo León
jorge.morenotr@uanl.edu.mx
https://orcid.org/0000-0002-5658-6763

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1. Introduction

According to a 2002 study by the Mexican Ministry of Health, 54.7 percent of Mexican households were not covered by any health insurance service. On the other hand, compared to their formal and insurance-covered counterparts, 42.6 percent were insured through the national social security system (Instituto Mexicano del Seguro Social, IMSS), 0.5 percent used private insurance companies, and only 1.5 percent were insured by both private and IMSS.

Motivated by these findings, in April 2003, the Mexican Congress approved a substantial reform for providing and financing public health services, the Seguro Popular (SP) program. However, in 2018, this policy was canceled by the federal government, and after a failed attempt to replace it, the new rules of health insurance remain unclear.

As a result of this cancellation, according to the National Council for the Evaluation of Social Development Policy (CONEVAL, 2022), the number of people without access to social security in 2022 reached 64.7 million. The organization explained that the number of Mexicans lacking access to health services increased from 20.1 million to 50.4 million, meaning that 39% of the population was unable to access adequate medical care. Can we estimate the gains and potential losses of this health insurance policy, using basic microeconomics and a risk analysis framework?

This paper presents a proposal for measuring the welfare gains from insurance in terms of the certainty equivalent under the particular case of normal truncated lotteries. Then, it uses this proposal for measuring the potential gains and losses from SP in Mexico. To pursue this purpose, it is proposed to use the direct utility value gains in consumption from a macro-calibration perspective applied in a partial equilibrium approach. Moreover, the model's empirical implementation uses the Mexican National Household Income Expenditure Survey (Encuesta Nacional de Ingreso y Gasto, ENIGH) for 2004. These surveys provide an excellent tool for measuring income and health expenditure and estimating the proposed gain measures ex ante of the policy implementation.

When a catastrophic health shock occurs, a lack of insurance translates into direct out-of-pocket expenditures for doctor visits, medications, and hospital services. Since a significant percentage of the people vulnerable to health income shocks concentrates on the lower income deciles, the potential gains and losses might differ across the income distribution. SP was public health insurance with two main objectives: 1) improve access to basic health services and financial protection against *out-of-pocket* expenditure in health services for the poorest population, and 2) change efficiency and equity in the public financing (subsidies) and the public provision of health services. The goal of SP was then to reach the Mexican population not covered by the conventional and formal social security network by the year 2010, and the target population includes households led by self-employed, unemployed, workers of the informal sector, or those outside the (formal) labor market.

2. Theoretical model

2.1. Household welfare and risk under censored shocks

2.1.1. Public policy evaluation: a general approach

The role of health as both an investment and consumption has been studied in detail in works collected by Murphy & Topel (2003). In all this literature, health is a consumption good that may provide utility either directly, by increasing the utility of other consumption goods, or by increasing the expected life of the agents. Murphy & Topel's estimations of welfare gains from health improvements in the USA during the 20th century indicate a substantial increase in consumer well-being resulting from improved health stocks and reduced mortality rates from various causes, including cardiovascular problems and cancer.

The framework proposed follows Lucas (2003) for measuring the losses derived from business cycles, applied in this case to a household environment with heterogeneity in their background characteristics.

Let us assume we are interested in comparing the effects of two policies, *N* and *I*. This case follows the implementation of the SP reform, while *N* the non-policy baseline stands for the baseline without policy intervention. Under *N* the household welfare is $U(C_N)$ where C_N is the level of consumption the household enjoys, and under the policy, *I* this welfare is given by $U(C_I)$ with the level of consumption C_I. Suppose that the consumer prefers *I* policy over *N*, so $U(C_I) > U(C_N)$, and let $k>0$ solve the following equality:

$$U(C_I) = U(C_N(1+k))$$

We call this number k - in units of percentage of all consumption goods - *the welfare gain* of a change in policy from *N* to *I*. Lucas concludes that by analyzing the policies in this way, we obtain a method that has both comprehensive units of measure and is built up from individual preferences.

2.1.2. Consumption and risk when health shocks are censored

Let us assume households' utility depends upon a composite level of consumption $\tilde{C}$, and they have a constant risk aversion utility function (Pratt, 1964) of the form:

$$U(\tilde{C}) = \alpha - \beta \exp\{-\theta\tilde{C}\}$$

with $\{\alpha, \beta, \theta\} \in R$, and $\beta, \theta\} \in R^{++}$ and where θ is the constant risk aversion coefficient of the household. For instance, a value of θ =0.005 implies that the individual is willing to pay up to 1.25% of their wealth to avoid a lottery with variance 5000.

When the lottery over an outcome $\tilde{C}$ has a known c.d.f. $F_C(\tilde{C})$ and utility is of the constant risk general form, then, following Deaton & Muellbauer (1980):

$$E[U(\tilde{C})] = \alpha - \beta m(-\gamma)$$

where $m(\cdot)$ is the moment-generating function of the random variable.

Let's assume the effective consumption of the household is the difference between its known level of income Y, which represents the gross potential for consumption, and a stochastic element representing *out-of-pocket* health expenditures.

$$\tilde{C} = Y - \tilde{E}$$

Health expenditure shock does not provide direct utility to households but reduces the possibilities of consuming other goods. Moreover, this expenditure is enforceable; therefore, the household derives utility only from net consumption after deducting the expenditure shock.

Assume the expenditure shock variable $\tilde{E}$ is linked to a normal distributed latent variable $\tilde{E}^*$ with support on the real numbers with mean μ and standard deviation σ^2 and ranks of value, which later may be household idiosyncratic.

Given the latent variable, $\tilde{E}_i^*$ is a censored normally distributed variable, then, $\tilde{C}$ has two components, one of which is normally distributed and censored.

In the absence of insurance coverage, which is the observed condition in which most low-income Mexican households live, the agents fully face the risk of health expenditures. Let us remember that for the case of a household with constant risk aversion utility that derives well-being from a normal lottery on a variable $\tilde{X}$ with mean m and standard deviation s^2 the certainty equivalent, $CE(\tilde{X})$, is given by definition:

$$U\left(CE(\tilde{X})\right) = EU(\tilde{X})$$

where:

$$CE(\tilde{X}) = m - \frac{1}{2}\theta s^2$$

$CE(\tilde{X})$ is the level of consumption under perfect certainty that leaves the household indifferent to having the lottery and facing uncertainty in the outcomes. Typically, the certainty equivalent of a normal lottery defined over the real space for a constant risk-averse agent depends upon three elements: mean, variance, and risk aversion. Indeed, under these conditions $CE(\tilde{X})$: i) is monotonically increasing in mean; ii) is monotonically decreasing in the variance (risk) of the lottery; and iii) has a linear trade-off between mean and variance in terms of the coefficient of risk aversion.

Normal-censored variables, combined with a constant risk-averse agent, provide a series of closed-form solutions to the certainty equivalent on lotteries defined over this family of random variables, depending on the nature of the censoring involved. The main results of these two elements prove helpful for this analysis and for later applications involving random variables with idiosyncratic censoring and different lottery support. The summary of these results is presented in Table 1 below, while the derivation of each term is available upon demand.

Table 1. Certainty equivalent on constant risk averse and normal censored lotteries[1]

Variable Support	Certainty Equivalent	M-likelihood term
$\tilde{X} \in [a, \infty)$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_1)$	$M_1 = \left(\frac{1 - \Phi\left(\frac{a - (\mu - \sigma^2\theta)}{\sigma}\right)}{1 - \Phi\left(\frac{a-\mu}{\sigma}\right)} \right)$
$\tilde{X} \in (-\infty, b]$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_2)$	$M_2 = \left(\frac{\Phi\left(\frac{b - (\mu - \sigma^2\theta)}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)} \right)$
$\tilde{X} \in [c, d], c < d$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_3)$	$M_3 = \left(\frac{\Phi\left(\frac{d - (\mu - \sigma^2\theta)}{\sigma}\right) - \Phi\left(\frac{c - (\mu - \sigma^2\theta)}{\sigma}\right)}{\Phi\left(\frac{d-\mu}{\sigma}\right) - \Phi\left(\frac{c-\mu}{\sigma}\right)} \right)$

Note*:* [1]Proofs available upon request.
Source: Author´s elaboration.

As Table 1 shows, censoring the support of the random variable has an additional effect on the certainty equivalent of the lottery, which depends on the type of censoring modeled. These properties play a key role in measuring the mean-variance potential tradeoff and welfare gains from insurance.

2.1.3. Willingness to pay for insurance

Following the same arguments as above, the willingness to pay is the maximum fee, F_T^{max}, which ensures the household is at least as well off as it would be without insurance.

For the case of total risk withdrawal, the willingness to pay F_T coincides with the following expression in terms of $CE(\tilde{C})$:

$$U\left(CE(\tilde{C})\right) = U\left(CE(Y - \tilde{E})\right) \leq U(Y - F_T)$$

$$F_T \leq CE(\tilde{E})$$

$$F_T^{max} = CE(\tilde{E}) = \mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}ln(M_3)$$

Moreover, as a proportion of the total income is:

$$\varphi_T^{max} \leq \frac{\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}ln(M_3)}{Y}$$

Hence, to estimate the welfare gains of insurance, we need the set of parameters $\Theta = \{Y, \theta, F, (\mu, \sigma^2)\}$.

3. Data

3.1 The Mexican databases

To evaluate the model's implications from an ex-ante perspective, I used the ENIGH 2004 survey to identify relevant variables related to income, expenditure, and context. I constructed deciles based on the quarterly income per capita of households, measuring access to health insurance by identifying all sources of aggregate household health insurance and considering all uninsured households where no member has access to health protection.

Using the ENIGH 2004, Table 2 reports income and health expenditure statistics by income decile, insurance status, and gender of the household head. These patterns offer insights into the distributional and gender-specific dimensions of economic vulnerability related to health spending by insurance coverage.

3.1.1. Income gradients and gender disparities

Income disparities between male- and female-headed households persist, particularly among the highest-income households. In decile 10, male-headed households report a mean quarterly income of US$4,215 (uninsured) and US$3,186 (insured), while female-headed households earn US$3,686 and US$2,979, respectively.

In contrast, income differences are negligible or even reversed in the lowest deciles. For example, in decile 1, female-headed uninsured households earn slightly more (US$121.4) than their male-headed counterparts (US$114.5).

3.1.2. Health spending and the role of insurance

Out-of-pocket health expenditures decrease as a share of income with rising deciles, consistent with standard consumption theory. Among uninsured households in decile 1, health spending comprises 6.6% of income for male-headed households and 7.6% for female-headed households. Insurance coverage reduces this burden to 4.7% and 2.9% for insured men and women, respectively.

In decile 4, uninsured male-headed households spend 4.6% of income on health, versus only 3.8% among their insured counterparts. For female-headed households in the same decile, the reduction is from 4.1% (uninsured) to 2.1% (insured). These reductions in expenditure shares are evident across all deciles, reinforcing the redistributive and consumption-smoothing role of public insurance.

3.1.3. Gendered vulnerability and variance in Expenditure

Despite similar or slightly lower average spending in absolute terms, female-headed households exhibit greater variability in both income and health spending. In decile 6, the standard deviation of health spending for uninsured female-headed households is US$92.5, compared to US$55.8 for male-headed households. This pattern holds especially among the uninsured.

Greater volatility in health expenditures suggests a higher probability of catastrophic health events, which disproportionately burden female-headed households.

3.1.4. Policy implications

The data provide compelling support for policy interventions aimed at expanding health insurance coverage, particularly among female-headed households in lower-income deciles. Insurance not only reduces mean health expenditures but also significantly compresses their variance, lowering exposure to income shocks and health-related poverty traps.

Moreover, the gender gaps in both income and expenditure volatility call for targeted redistributive tools. Female-headed households, especially when uninsured, face a compounded disadvantage: lower and more variable income, coupled with more volatile health spending. The following section presents the calibration of the model.

4. Results and discussion

4.1. Empirical strategy: from model to data and calibration results

This section presents a calibration exercise of the welfare gains using the observed behavior from the Mexican data sets described before.

Table 3 presents estimated welfare gains, expressed as the willingness to pay (WTP) for actuarially fair health insurance, under various combinations of expected medical expenses (mean) and associated risk (variance), for a household with a quarterly income of US$200. The estimates are computed under a constant absolute risk aversion utility function with a coefficient equal to 1, implying a large aversion toward health risk[1]. Additionally, I considered the case of insurance with full-risk withdrawal for the case where households face a zero fee.

[1] Viscusi (1991) proposes values lower than 1 for health and life insurance.

Table 2: Income and health expenditure by decile, insurance status, and gender (US dollars per quarter in 2004 value)

Decile	INCOME				HEALTH EXPENDITURE				HEALTH EXPENDITURE (%)			
	Uninsured		Insured		Uninsured		Insured		Uninsured		Insured	
	Men	Women	Men	Women	Men	Women	Men	Women	Men	Women	Men	Women
1	114.5	121.4	138.3	133.0	7.4	8.5	7.1	3.8	6.6	7.6	4.7	2.9
	36.5	*33.1*	*29.7*	*22.6*	*16.8*	*10.5*	*15.2*	*2.8*	*14.1*	*10.5*	*9.3*	*2.4*
2	205.3	207.7	206.4	211.2	12.6	12.8	6.9	4.0	6.2	6.0	3.5	1.9
	21.5	*21.0*	*21.5*	*18.1*	*27.8*	*16.0*	*16.8*	*4.9*	*13.6*	*7.6*	*9.2*	*2.4*
3	276.0	273.5	278.0	275.7	12.4	11.2	7.2	14.8	4.5	4.0	2.6	5.6
	20.8	*19.3*	*19.8*	*21.1*	*26.6*	*18.1*	*12.3*	*22.4*	*9.5*	*6.2*	*4.6*	*8.6*
4	348.9	349.1	349.2	353.8	16.0	14.5	13.4	7.2	4.6	4.1	3.8	2.1
	22.7	*22.1*	*21.6*	*22.2*	*31.0*	*21.2*	*27.2*	*15.8*	*8.9*	*5.8*	*7.6*	*4.6*
5	429.1	428.1	433.6	427.1	20.9	15.8	15.0	14.1	4.9	3.7	3.5	3.2
	25.3	*26.6*	*25.4*	*23.8*	*37.0*	*27.2*	*46.2*	*21.5*	*8.7*	*6.3*	*11.2*	*4.8*
6	536.8	534.1	534.0	536.8	27.1	42.4	13.7	19.0	5.0	8.2	2.6	3.5
	34.3	*35.1*	*36.3*	*32.4*	*55.8*	*92.5*	*29.5*	*48.1*	*10.0*	*18.4*	*5.5*	*8.9*
7	667.6	685.0	675.4	678.2	29.7	33.6	20.3	20.4	4.4	4.8	3.0	3.0
	43.6	*43.2*	*43.6*	*39.1*	*59.4*	*56.8*	*37.6*	*29.4*	*9.0*	*8.0*	*5.3*	*4.4*
8	889.2	898.6	890.9	904.0	45.0	42.0	28.2	30.3	5.1	4.7	3.2	3.4
	92.0	*83.7*	*85.4*	*87.2*	*90.0*	*71.8*	*76.7*	*53.4*	*10.1*	*8.2*	*8.7*	*6.1*
9	1297.4	1288.1	1302.1	1299.9	57.1	54.7	39.4	53.8	4.6	4.2	3.1	4.1
	161.4	*172.8*	*156.6*	*163.7*	*122.6*	*68.8*	*72.9*	*86.2*	*10.2*	*5.4*	*6.0*	*6.4*
10	4215.6	3686.0	3186.4	2979.2	172.5	133.7	76.6	102.9	3.4	3.8	2.5	3.7
	6062.2	*8488.7*	*2798.9*	*1538.0*	*651.1*	*298.1*	*191.2*	*261.3*	*5.3*	*5.2*	*5.2*	*11.2*
Total	668.5	815.7	1066.4	1133.8	668.5	815.7	1066.4	1133.8	5.1	5.1	3.0	3.5
	1765.2	*2877.0*	*1474.9*	*1144.0*	*1765.2*	*2877.0*	*1474.9*	*1144.0*	*10.8*	*9.0*	*7.2*	*7.4*

Note: Mean in regular font, *Standard Deviation in italics*.
Source: Author´s elaboration with data from ENIGH 2004, INEGI Mexico.

Welfare gains increase monotonically with the expected value of losses but exhibit a nonlinear sensitivity to risk as variance increases. At low levels of expected expenses (e.g., US$3 or US$8), changes in variance have a minimal impact on WTP. For instance, at a mean of US$3, welfare gains increase marginally from 1.1% to 1.3% as variance rises from 3 to 72. This flat response is attributable to the truncation at zero, which limits risk at low mean values.

In contrast, for higher expected losses, variance plays a prominent role. At a mean of US$25, WTP drops significantly from 13.3% to 3.6% as variance rises from 3 to 72, reflecting the convexity of the utility loss under greater uncertainty. However, the marginal impact of variance is non-constant, with the largest welfare reduction between variances of 3 and 24.

Table 3: Welfare gains for different mean-variance health shock specifications, household quarterly income US$200.

Mean	Risk Aversion Coefficient = 1 Variance				
	3	**15**	**24**	**42**	**72**
3	1.1%	1.1%	1.2%	1.2%	1.3%
8	3.4%	1.9%	1.8%	1.7%	1.6%
12	5.5%	3.1%	2.5%	2.1%	1.9%
20	10.2%	6.7%	5.0%	3.6%	2.8%
25	13.3%	9.6%	7.3%	5.0%	3.6%
50	32.0%	27.0%	23.5%	17.0%	10.7%

Source: Author's estimations.

The nonlinear structure of these results underscores that the marginal value of insurance coverage depends not only on the level of risk but also on its interaction with the size of the expected financial burden.

These results have policy implications for targeting subsidies or designing insurance products in low-income settings, where both the mean and variance of out-of-pocket shocks are tightly bounded. Also, these results represent upper bounds for the gains from risk reduction, particularly because they assume a total withdrawal of risk and that households do not protect themselves using other portfolio options to diversify their risk in the absence of insurance. Nonetheless, the complementarity of the proposal above with other approaches, such as natural experiments or dynamic general equilibrium, would provide a more comprehensive evaluation of the welfare implications of insurance.

5. Conclusion and recommendations

This paper presents a proposal for measuring the distribution of gains among Mexican households resulting from the expansion of welfare health insurance coverage. The analysis uses a structural model that measures the value of gains from reducing risk on a household's consumption.

Using data from ENIGH 2004 to capture the ex-ante value of SP insurance, I presented evidence that suggests significant differences in the relative risk of consumption between insured and uninsured households within and across deciles. In particular, the data show that households led by women at the bottom of the income distribution are relatively more vulnerable to positive health out-of-pocket expenditure shocks, and those shocks present a higher mean and variance when measured as a percentage of their income.

To conclude, this paper is a first step into a more formal analysis of the implications of health welfare policies. Further analysis implemented in future research and linked to this work would provide a better understanding of the effects of this policy change under different model specifications, considering, for instance, a natural experiment approach or a general equilibrium framework. This additional work would provide a more complete evaluation of the welfare effects of this type of public policy.

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